VERIFICATION OF SOLUTIONS OF DIFFERENTIAL EQUATIONS

1. Verify that the function $y = x^4$ satisfies the differential equation

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0.$$

2. Verify that the function $y = Ae^{3x} + Be^{x}$, where A and B are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0.$$

3. Verify that the function $y = Ae^{x} + Be^{2x}$, where A and B are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0.$$

4. (a) Verify that the function $y = A\sin 2x + B\cos 2x$, where A and B are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} + 4y = 0.$$

(b) Verify that, in general, the function $y = A\sin kx + B\cos kx$, where A, B and k are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} + k^2y = 0.$$

5. (a) Verify that the function $y = Ae^{3x} + Be^{-3x}$, where A and B are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} - 9y = 0.$$

(b) Verify that, in general, the function $y = Ae^{kx} + Be^{-kx}$, where A, B and k are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} - k^2y = 0.$$

6. Verify that the function $y = x^n$, where n is a constant, satisfies the differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0.$$

7. Verify that the function $y = Ae^{5x} + Be^{-3x} - \frac{1}{7}e^{4x}$, where A and B are constants, satisfies the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 15y = e^{4x}.$$

8. Verify that the function $y = e^{x}\sin x$ satisfies the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

9. Verify that the function $y = x^3\ln x$ satisfies the differential equation

$$x^2\frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 9y = 0.$$

10. Verify that the function $y = e^{-x}\cos 2x$ satisfies the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0.$$

DIFFERENTIAL EQUATIONS 1

1. Find the general solution of each differential equation below.

(a) $\frac{dy}{dx} = \frac{x}{2y}$ (b) $\frac{dy}{dx} = \frac{x}{y^2}$

(c) $\frac{dy}{dx} = \frac{x^3}{y}$ (d) $\frac{dy}{dx} = \frac{x+2}{y}$

(e) $2y\frac{dy}{dx} = 3x^2$ (f) $y^2\frac{dy}{dx} = x^5$

(g) $\frac{dy}{dx} = \frac{2}{\cos y}$ (h) $\frac{dy}{dx} = \frac{\cos x}{y}$

(i) $\sin y\frac{dy}{dx} = 1$ (j) $\cos 2y\frac{dy}{dx} - x = 0$

(k) $\frac{dy}{dx} = \frac{4x}{e^y}$ (l) $\frac{dy}{dx} = \frac{e^x}{y}$

(m) $e^y\frac{dy}{dx} = 1$ (n) $\frac{dy}{dx} = \frac{x}{e^{2y}}$

(o) $3y\frac{dy}{dx} = 5x^2$ (p) $x + 4y\frac{dy}{dx} = 0$

(q) $y^2\frac{dy}{dx} = x + 1$ (r) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$

(s) $\frac{dy}{dx} = \frac{x^2 - 1}{y^3}$ (t) $x^2 + 3y\frac{dy}{dx} = 0$

(u) $\frac{dy}{dx} = 4x\sqrt{y}$ (v) $e^{4y}\frac{dy}{dx} = x^2(x+1)$

(w) $y\frac{dy}{dx} = \frac{1}{\sqrt{x}}$ (x) $1 + y\frac{dy}{dx} = x$

(y) $y\frac{dy}{dx} = \sqrt{x}$ (z) $\sin y\frac{dy}{dx} = e^{2x}$

2. Find the general solution of each differential equation below.

(a) $\frac{dy}{dx} = 4x\sqrt{1-y^2}$ (b) $\frac{dy}{dx} = 2x(1+y^2)$

(c) $\frac{dy}{dx} = 2x\sqrt{4-y^2}$ (d) $y\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

(e) $(1+x^2)y^2\frac{dy}{dx} = 1$ (f) $\frac{dy}{dx} = x(4+y^2)$

3. (a) Use integration by parts to find $\int xe^x\,dx$.

(b) Hence find the general solution of the differential equation $\frac{dy}{dx} = \frac{xe^x}{y}$.

4. (a) Use integration by parts to find $\int x\sin x\,dx$.

(b) Hence find the general solution of the differential equation $\frac{dy}{dx} = \frac{x\sin x}{e^y}$.

5. Study the worked example below carefully.

Worked Example

Find the general solution of the differential equation

$$\frac{dy}{dx} = y^2\sin x.$$

Solution

$$\frac{dy}{dx} = y^2\sin x \quad\Rightarrow\quad dy = y^2\sin x\,dx$$

$$\Rightarrow\quad \int\frac{1}{y^2}dy = \int\sin x\,dx$$

$$\Rightarrow\quad \int y^{-2}dy = \int\sin x\,dx$$

$$\Rightarrow\quad \frac{1}{(-1)}y^{-1} = -\cos x + C$$

$$\Rightarrow\quad -\frac{1}{y} = -\cos x + C \qquad \{\times(-1)\}$$

$$\Rightarrow\quad \frac{1}{y} = \cos x + C$$

$$\Rightarrow\quad y = \frac{1}{\cos x + C}$$

(a) Show that the general solution of the differential equation

$$\frac{dy}{dx} = e^x y^2$$

can be expressed in the form $y = \frac{1}{C - e^x}$, where C is a constant.

(b) Show that the general solution of the differential equation

$$x^5 \frac{dy}{dx} = 4y^2$$

can be expressed in the form $y = \frac{x^4}{1 + Cx^4}$, where C is a constant.

ANSWERS

1. (a) $2y^2 = x^2 + C$ (b) $2y^3 = 3x^2 + C$ (c) $2y^2 = x^4 + C$
   (d) $y^2 = x^2 + 4x + C$ (e) $y^2 = x^3 + C$ (f) $2y^3 = x^6 + C$
   (g) $\sin y = 2x + C$ (h) $y^2 = 2\sin x + C$ (i) $\cos y = C - x$
   (j) $\sin 2y = x^2 + C$ (k) $y = \ln(2x^2 + C)$ (l) $y^2 = 2e^x + C$
   (m) $y = \ln(x + C)$ (n) $y = \frac{1}{2}\ln(x^2 + C)$ (o) $9y^2 = 10x^3 + C$
   (p) $4y^2 = C - x^2$ (q) $2y^3 = 3x^2 + 6x + C$ (r) $\sin y = C - \cos x$
   (s) $3y^4 = 4x^3 - 12x + C$ (t) $9y^2 = C - 2x^3$ (u) $y = (x^2 + C)^2$
   (v) $y = \frac{1}{4}\ln(x^4 + 2x^2 + C)$ (w) $y^2 = 4\sqrt{x} + C$ (x) $y^2 = x^2 - 2x + C$
   (y) $3y^2 = 4x^{\frac{3}{2}} + C$ (z) $2\cos y = C - e^{2x}$

2. (a) $y = \sin(2x^2 + C)$ (b) $y = \tan(x^2 + C)$ (c) $y = 2\sin(x^2 + C)$
   (d) $y^2 = 2\sin^{-1} x + C$ (e) $y^3 = 3\tan^{-1} x + C$ (f) $y = 2\tan(x^2 + C)$

3. (a) $\int xe^x dx = xe^x - e^x + C$ (b) $y^2 = 2(x-1)e^x + C$

4. (a) $\int x\sin x dx = -x\cos x + \sin x + C$ (b) $y = \ln(\sin x - x\cos x + C)$