Practice Examination C (Unit 3)

(Assessing only Unit 3)

MATHEMATICS

Advanced Higher Grade

Time allowed - 1 hour

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used in this paper.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (*a*) Find the first five terms of the Maclaurin Series for $(1+2x)^{3/2}$. (4)

   (*b*) For what values of x is this series valid? (2)

   (*c*) Use this expansion to find an approximation for $1\cdot4^{3/2}$ to 4 decimal places. (2)

2. For what value(s) of x, $(x \neq 0)$, is the matrix $\begin{pmatrix} x & 2x \\ x-1 & 3x-5 \end{pmatrix}$ singular? (3)

3. Find the g.c.d. of 320 and 153.

   Hence find a, b such that $320a + 153b = 1$. (5)

4. Show that the vectors $\underline{a} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$, $\underline{b} = \begin{pmatrix} 6 \\ -2 \\ 8 \end{pmatrix}$ and $\underline{c} = \begin{pmatrix} -5 \\ 5 \\ 4 \end{pmatrix}$ are coplanar and find the equation of the plane.

   Hence find the angle between this plane and the plane $2x + 3z = 5$ (7)

5. A recurrence relation is given by the formula

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{3}{x_n}\right).$$

   Find the fixed points of this recurrence relation and hence show that the series converges for $x_0 = 0\cdot5$, stating the exact value of the root to which it converges. (5)

6. Prove that the differential equation $x\,dy + 2y\,dx = \cos x\,dx$ is linear and hence solve the equation given that $y(\pi) = 1$. (10)

END OF QUESTION PAPER

Marking Scheme - AH – Practice Examination C (Unit 3)

	Give one mark for each •	Illustrations for awarding each mark
1(a)	**ans:** $1+3x+\frac{3x^2}{2}-\frac{x^3}{2}+\frac{3x^4}{8}$ **4 marks** • finds values for $f(0)$ and $f'(0)$ • finds values for $f''(0)$, $f'''(0)$ and $f^{iv}(0)$ • using correct series • final statement of 5 terms	• $f(0)=1;\ f'(0)=3$ • $f''(0)=3;\ f'''(0)=-3;\ f^{iv}(0)=9$ • $f(x)\approx f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3$ • $1+3x+\frac{3x^2}{2}-\frac{x^3}{2}+\frac{3x^4}{8}$
1(b)	**ans:** $\lvert x\rvert<\frac{1}{2}$ **2 marks** • knows how to find range of validity • finds range correctly	• $\lvert 2x\rvert<1$ • $\lvert x\rvert<\frac{1}{2}$
1(c)	**ans:** 1·6566 **2 marks** • knows to substitute 0·2 for x • finding correct approximation	• $1+3(0\cdot 2)+\frac{3(0\cdot 2)^2}{2}-\frac{(0\cdot 2)^3}{2}+\frac{3(0\cdot 2)^4}{8}$ • 1·6566
2.	**ans:** $x=3$ **3 marks** • knows to find determinant • knows to put determinant equal to zero • finds value of x	• $x(3x-5)-2x(x-1)$ • $x(3x-5)-2x(x-1)=0$ • $x=0,\ x=3$ but $x\neq 0$
3.	**ans:** $a=11,\ b=-23$ **5 marks** • uses correct algorithm • correct steps in algorithm • final statement • method • answer	• 320 = 2 × 153 + 14 • 153 = 10 × 14 + 13 14 = 1 × 13 + 1 13 = 13 × 1 • gcd of 320 and 153 = 1 • 1 = 14 – 1 × 13 = 14 – 1 × (153 – 10 × 14) = 11 × 14 – 153 = 11 × (320 – 2 × 153) – 153 = 11 × 320 – 23 × 153 • $a=11,\ b=-23$

	Give one mark for each •	Illustrations for awarding each mark
4.	**ans:** proof, $12x + 16y - 5z = 0$, 83^{o} **7 marks**	
	• method	• Suppose $\underline{c} = \lambda\underline{a} + \mu\underline{b}$ then $\begin{pmatrix} -5 \\ 5 \\ 4 \end{pmatrix} = \lambda\begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix} + \mu\begin{pmatrix} 6 \\ -2 \\ 8 \end{pmatrix}$
	• knows how to find unknowns	$-5 = -\lambda + 6\mu$ • $5 = 2\lambda - 2\mu$ $4 = 4\lambda + 8\mu$
	• finds unknowns and checks with z coordinate	• $\lambda = 2,\ \mu = -\frac{1}{2} \Rightarrow 4 = 4 \times 2 + 8 \times \frac{1}{2}$ - yes! $\underline{c} = 2\underline{a} - \frac{1}{2}\underline{b}$ so vectors are coplanar
	• knows how to find normal	• $\overrightarrow{ab} \times \overrightarrow{ac} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 7 & -4 & 4 \\ -4 & 3 & 0 \end{vmatrix}$
	• finds equation of plane	• $-12x - 16y + 5z = 0$ or $12x + 16y - 5z = 0$
	• knows how to find angle	• $\cos\theta = \frac{\underline{a} \cdot \underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert} = \frac{\begin{pmatrix} 12 \\ 16 \\ -5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}}{\sqrt{425} \cdot \sqrt{13}} = \frac{9}{74.3303}$
	• finds angle correctly	• $\theta = 83 \cdot 0^{o}$
5.	**ans:** $x = \pm\sqrt{3}$, $\sqrt{3}$ **5 marks**	
	• knows how to find fixed points	• $x = \frac{1}{2}\left(x + \frac{3}{x}\right)$
	• correct steps in solving	• $2x = x + \frac{3}{x} \Rightarrow x^2 = 3$
	• correct fixed points	• $x = \pm\sqrt{3}$
	• correct method for finding root	• $x_1 = \frac{1}{2}\left(0 \cdot 5 + \frac{3}{0 \cdot 5}\right) = 3 \cdot 25$; $x_2 = 2 \cdot 0865;\ x_3 = 1 \cdot 7622;\ x_4 = 1 \cdot 7323$
	• states exact value of root	• converges to $\sqrt{3}$

	Give one mark for each •	Illustrations for awarding each mark
6.	**ans:** $y = \frac{\sin x}{x} + \frac{\cos x + \pi^2 + 1}{x^2}$ **10 marks**	
	• knows to rearrange equation	•
	• rearranges equation correctly	• $\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x}$
	• states form of linear equation	• $\frac{dy}{dx} + P(x)y = Q(x)$
	• finds integrating factor	• I. F. $= e^{\int \frac{2}{x} dx} = e^{2\ln\lvert x\rvert} = x^2$
	• states modified equation	• $\frac{d}{dx}\left(x^2 y\right) = x\cos x$
	• re-writes as an integral	• $x^2 y = \int x\cos x\, dx$
	• integration by parts	• $= x\sin x - \int \sin x\, dx = x\sin x + \cos x + C$
	• final form of answer	• $y = \frac{\sin x}{x} + \frac{\cos x}{x^2} + \frac{C}{x^2}$
	• finds constant of integration	• $C = \pi^2 + 1$
	• states exact solution	• $y = \frac{\sin x}{x} + \frac{\cos x + \pi^2 + 1}{x^2}$

TOTAL 38 MARKS