Practice Examination A (Unit 3)

(Assessing only Unit 3)

MATHEMATICS

Advanced Higher Grade

Time allowed - 1 hour

Read Carefully

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used in this paper.**
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

All questions should be attempted

1. (*a*) Find the first four terms of the Maclaurin Series for $(1+2x)^{3/2}$. (4)

   (*b*) For what values of *x* is this series valid? (2)

   (*c*) Use this expansion to find an approximation for $1\cdot2^{3/2}$ to 4 decimal places. (3)

2. Find values of *h* and *k* for which the system

$$\begin{aligned}2x + hy &= 8\\ x + 3y &= k\end{aligned}$$

has (*a*) infinitely many solutions; (*b*) no solution, giving a reason for each answer. (6)

3. (*a*) Use the vector product to calculate the area of a triangle with vertices P(2, -1, 0), Q(1, 1, -1) and R(3, 4, 2). (4)

   (*b*) Find the equation of the plane passing through triangle PQR. (3)

   (*c*) Determine the point of intersection of the line

$$\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{3}$$

with this plane. (4)

4. (*a*) Show that the matrix $\boldsymbol{A} = \begin{bmatrix} 3 & -1 & -4 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ is invertible. (3)

   (*b*) Use elementary row operations to find $\boldsymbol{A}^{-1}$. (5)

   (*c*) *Hence* solve the system of equations

$$\begin{aligned}3x - y - 4z &= -1\\ 2x + y \quad\;\; &= 9\\ y + 2z &= 7\end{aligned}$$

(2)

Marking Scheme - AH – Practice Examination A (Unit 3)

	Give one mark for each •	Illustrations for awarding each mark
1(a)	**ans:** $1+3x+\frac{3x^2}{2}-\frac{x^3}{2}$ **4 marks** • finds values for $f(0)$ and $f'(0)$ • finds values for $f''(0)$ and $f'''(0)$ • using correct series • final statement of 4 terms	 • $f(0)=1;\ f'(0)=3$ • $f''(0)=3;\ f'''(0)=-3$ • $f(x)\approx f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3$ • $1+3x+\frac{3x^2}{2}-\frac{x^3}{2}$
1(b)	**ans:** $\lvert x\rvert < \frac{1}{2}$ **2 marks** • knows how to find range of validity • finds range correctly	 • $\lvert 2x\rvert < 1$ • $\lvert x\rvert < \frac{1}{2}$
1(c)	**ans:** 1·3145 **3 marks** • knows to substitute 0·1 for x • substituting correctly • finding correct approximation	 • • $1+3(0\cdot1)+\frac{3(0\cdot1)^2}{2}-\frac{(0\cdot1)^3}{2}$ • 1·3145
2.	**ans:** $h=6,\ k=4$; $h=6,\ k\neq4$ **6 marks** • constructs augmented matrix • uses row reductions to reduce system to upper triangular form • correct values of h and k for (a) • correct reasoning for (a) • correct values of h and k for (b) • correct reasoning for (b)	 • $\left[\begin{array}{cc\|c} 2 & h & 8 \\ 1 & 3 & k \end{array}\right] \rightarrow \left[\begin{array}{ccc} 1 & 3 & k \\ 2 & h & 8 \end{array}\right]$ • $\left[\begin{array}{cc\|c} 1 & 3 & k \\ 0 & h-6 & 8-2k \end{array}\right]$ • $h=6,\ k=4$ • solution is $x=4-3y$ i.e. infinitely many • $h=6,\ k\neq4$ • bottom line reads $0=8-2k$ which is not possible since $8-2k\neq 0$

	Give one mark for each •	Illustrations for awarding each mark
3(a)	**ans:** $\frac{5}{2}\sqrt{3}$ **4 marks** • finding direction vectors • using vector product correctly • knows how to find area of triangle • finding area	• $\overrightarrow{PQ} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}; \overrightarrow{PR} = \begin{pmatrix}1\\5\\2\end{pmatrix}$ • $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix}\underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k}\\-1 & 2 & -1\\1 & 5 & 2\end{vmatrix} = 9\underset{\sim}{i} + \underset{\sim}{j} - 7\underset{\sim}{k}$ • Area = $\frac{1}{2}\left\|9\underset{\sim}{i} + \underset{\sim}{j} - 7\underset{\sim}{k}\right\|$ • $\frac{1}{2}\sqrt{75} = \frac{5}{2}\sqrt{3}$
3(b)	**ans:** $9x + y - 7z = 17$ **3 marks** • know to use vector product for normal • use coefficients of normal for equation of plane • find value of k	• $\underset{\sim}{n} = 9\underset{\sim}{i} + \underset{\sim}{j} - 7\underset{\sim}{k}$ • $9x + y - 7z = k$ • $9(2) + (-1) - 7(0) = k$; $k = 17$
3(c)	**ans:** (-15, -9, -23) **4 marks** • write equation of line in parametric form • sub parametric equations into equation of plane • find parameter t • find point of intersection	• $x = 1 + 2t;\ y = -1 + t;\ z = 1 + 3t$ • $9(1 + 2t) + (-1 + t) - 7(1 + 3t) = 17$ • $t = -8$ • $x = 1 + 2(-8) = -15$; $y = -1 + (-8) = -9$; $z = 1 + 3(-8) = -23$
4(a)	**ans:** Proof: show determinant $\neq 0$ **3 marks** • know to find determinant • find determinant • conclusion	• det $\boldsymbol{A} = \begin{vmatrix}3 & -1 & -4\\2 & 1 & 0\\0 & 1 & 2\end{vmatrix}$ • 2 • A is invertible as det $\boldsymbol{A} \neq 0$

	Give one mark for each •	Illustrations for awarding each mark
4(b)	**ans:** $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -4 \\ 1 & -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$ **5 marks**	
	• construct augmented matrix with ***I*** on RHS	• $\left[\begin{array}{ccc:ccc} 3 & -1 & -4 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$
	• row reductions to upper triangular form	• $\left[\begin{array}{ccc:ccc} 3 & -1 & -4 & 1 & 0 & 0 \\ 0 & \frac{5}{3} & \frac{8}{3} & -\frac{2}{3} & 1 & 0 \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} & 1 \end{array}\right]$
	• non-zero entries of LHS in leading diagonal only	• $\left[\begin{array}{ccc:ccc} 3 & 0 & 0 & 3 & -3 & 6 \\ 0 & \frac{5}{3} & 0 & -\frac{10}{3} & 5 & -\frac{20}{3} \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} & -\frac{3}{5} & 1 \end{array}\right]$
	• makes LHS = ***I***	• $\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & -2 & 3 & -4 \\ 0 & 0 & 1 & 1 & -\frac{3}{2} & \frac{5}{2} \end{array}\right]$
	• find inverse matrix	• $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -4 \\ 1 & -\frac{3}{2} & \frac{5}{2} \end{bmatrix}$
4(c)	**ans:** (4, 1, 3) **2 marks**	
	• premultiply by inverse matrix	• $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 3 & -4 \\ 1 & -\frac{3}{2} & \frac{5}{2} \end{bmatrix}\begin{bmatrix} -1 \\ 9 \\ 7 \end{bmatrix}$
	• find solution	• (4, 1, 3)

Total 36 Marks