SECOND-ORDER LINEAR DIFFERENTIAL EQUATIONS (HOMOGENEOUS)

1. Find the general solution of each differential equation.

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$ (b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$

(c) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ (d) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

(e) $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$ (f) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$

(g) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$ (h) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$

(i) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$ (j) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

(k) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ (l) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$

(m) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$ (n) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$

(o) $\frac{d^2y}{dx^2} - 16y = 0$ (p) $2\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 3y = 0$

(q) $9\frac{d^2y}{dx^2} - 30\frac{dy}{dx} + 25y = 0$ (r) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$

(s) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (t) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$

(u) $2\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ (v) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = 0$

(w) $4\frac{d^2y}{dx^2} - 9y = 0$ (x) $9\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 4y = 0$

(y) $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$ (z) $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 5y = 0$

2. Find the particular solution of each differential equation subject to the conditions given in brackets.

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ (when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 2$)

(b) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ (when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 10$)

(c) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$ (when $x = 0$, $y = 3$ and $\frac{dy}{dx} = 5$)

(d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ (when $x = 0$, $y = 6$ and $\frac{dy}{dx} = 0$)

(e) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ (when $x = 0$, $y = 2$ and $\frac{dy}{dx} = 9$)

(f) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ (when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$)

(g) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$ (when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 3$)

(h) $\frac{d^2y}{dx^2} - y = 0$ (when $x = 0$, $y = 1$ and $\frac{dy}{dx} = 2$)

(i) $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$ (when $x = 0$, $y = 3$ and $\frac{dy}{dx} = 20$)

(j) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 10y = 0$ (when $x = 0$, $y = 2$ and $\frac{dy}{dx} = 1$)

(k) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0$ (when $x = 0$, $y = 5$ and $\frac{dy}{dx} = 2$)

(l) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ (when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 3$)

(m) $\frac{d^2y}{dx^2} + 9y = 0$ (when $x = 0$, $y = 10$ and $\frac{dy}{dx} = 0$)

(n) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ (when $x = 0$, $y = 2$ and $\frac{dy}{dx} = 3$)

(o) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0$ (when $x=0,\ y=\frac{1}{4}$ and $\frac{dy}{dx}=-1$)

(p) $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+13y=0$ (when $x=0,\ y=2$ and $\frac{dy}{dx}=0$)

(q) $2\frac{d^2y}{dx^2}+7\frac{dy}{dx}-4y=0$ (when $x=0,\ y=1$ and $\frac{dy}{dx}=2$)

(r) $2\frac{d^2y}{dx^2}-6\frac{dy}{dx}+5y=0$ (when $x=0,\ y=3$ and $\frac{dy}{dx}=-2$)

(s) $\frac{d^2y}{dx^2}-10\frac{dy}{dx}+25y=0$ (when $x=0,\ y=1$ and $\frac{dy}{dx}=2$)

(t) $2\frac{d^2y}{dx^2}+5\frac{dy}{dx}-3y=0$ (when $x=0,\ y=1$ and $\frac{dy}{dx}=3$)

ANSWERS

1. (a) $y=Ae^{x}+Be^{3x}$ (b) $y=Ae^{3x}+Be^{-2x}$

(c) $y=Ae^{2x}+Be^{-x}$ (d) $y=(Ax+B)e^{2x}$

(e) $y=(Ax+B)e^{5x}$ (f) $y=(Ax+B)e^{-3x}$

(g) $y=e^{x}(A\sin 2x+B\cos 2x)$ (h) $y=e^{3x}(A\sin 2x+B\cos 2x)$

(i) $y=e^{-2x}(A\sin x+B\cos x)$ (j) $y=Ae^{2x}+Be^{3x}$

(k) $y=A+Be^{2x}$ (l) $y=(Ax+B)e^{4x}$

(m) $y=e^{-2x}(A\sin 3x+B\cos 3x)$ (n) $y=Ae^{-2x}+Be^{-3x}$

(o) $y=Ae^{4x}+Be^{-4x}$ (p) $y=Ae^{\frac{1}{2}x}+Be^{-3x}$

(q) $y=(Ax+B)e^{\frac{5}{3}x}$ (r) $y=e^{2x}(A\sin 2x+B\cos 2x)$

(s) $y=e^{-x}(A\sin x+B\cos x)$ (t) $y=Ae^{3x}+Be^{4x}$

(u) $y=Ae^{\frac{3}{2}x}+Be^{-2x}$ (v) $y=e^{-2x}(A\sin 2x+B\cos 2x)$

(w) $y=Ae^{\frac{3}{2}x}+Be^{-\frac{3}{2}x}$ (x) $y=(Ax+B)e^{-\frac{2}{3}x}$

(y) $y=e^{\frac{1}{2}x}\left(A\sin\frac{3}{2}x+B\cos\frac{3}{2}x\right)$ (z) $y=Ae^{-\frac{1}{3}x}+Be^{-\frac{5}{2}x}$

2. (a) $y=2e^{2x}-2e^{x}$ (b) $y=2e^{2x}-2e^{-3x}$

(c) $y=2e^{x}+e^{3x}$ (d) $y=4e^{x}+2e^{-2x}$

(e) $y=(3x+2)e^{3x}$ (f) $y=(4x+1)e^{-2x}$

(g) $y=e^{x}(2\sin x+\cos x)$ (h) $y=2e^{x}-1$

(i) $y=(5x+3)e^{5x}$ (j) $y=e^{3x}(-5\sin x+2\cos x)$

(k) $y=2e^{4x}+3e^{-2x}$ (l) $y=3xe^{2x}$

(m) $y=10\cos 3x$ (n) $y=e^{-x}(5\sin x+2\cos x)$

(o) $y=\left(-\frac{3}{4}x+\frac{1}{4}\right)e^{-x}$ (p) $y=e^{-2x}\left(\frac{4}{3}\sin 3x+2\cos 3x\right)$

(q) $y=\frac{4}{3}e^{\frac{1}{2}x}-\frac{1}{3}e^{-4x}$ (r) $y=e^{\frac{3}{2}x}\left(-13\sin\frac{1}{2}x+3\cos\frac{1}{2}x\right)$

(s) $y=(-3x+1)e^{5x}$ (t) $y=\frac{12}{7}e^{\frac{1}{2}x}-\frac{5}{7}e^{-3x}$

NON-HOMOGENEOUS SECOND-ORDER LINEAR DIFFERENTIAL EQUATIONS

1. Find the general solution of each differential equation (the particular integral will be a polynomial function in each case).

(a) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 6x + 1$

(b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 12x$

(c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 8x^2 + 3$

(d) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 20x + 4$

(e) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 18$

(f) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 12x + 3$

(g) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 8 - 10x$

(h) $\frac{d^2y}{dx^2} + 4y = 3x$

(i) $\frac{d^2y}{dx^2} + 16y = 4x^2$

(j) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = x$

(k) $4\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 2y = 1 - x^2$

2. Find the general solution of each differential equation (the particular integral will be an exponential function in each case).

(a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4e^{2x}$

(b) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8e^{3x}$

(c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 10e^{2x}$

(d) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4e^{-x}$

(e) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{4x}$

(f) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 10e^{3x}$

(g) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = e^{-2x}$

(h) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^{-x}$

(i) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = e^{2x}$

(j) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 10e^{x}$

(k) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-3x}$

3. Find the general solution of each differential equation (the particular integral will be a trigonometric function in each case).

(a) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 40\sin x$

(b) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 3\sin x + 19\cos x$

(c) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 25\sin x$

(d) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = \sin x$

(e) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 5\sin x$

(f) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = \sin 2x$

(g) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = \sin x$

(h) $2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 2\cos x$

***4.** Find the general solution of each differential equation (the particular solution will be an amended exponential function in each case).

(a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{x}$

(b) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 3e^{2x}$

(c) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 4e^{3x}$

(d) $\frac{d^2y}{dx^2} + 7\frac{dy}{dx} + 12y = e^{-3x}$

(e) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 6e^{4x}$

ANSWERS

1. (a) $y = Ae^{2x} + Be^{3x} + x + 1$ (b) $y = Ae^{2x} + Be^{-x} - 6x + 3$

(c) $y = Ae^{x} + Be^{4x} + 2x^2 + 5x + 6$ (d) $y = Ae^{2x} + Be^{-5x} - 2x - 1$

(e) $y = (Ax + B)e^{3x} + 2$ (f) $y = Ae^{4x} + Be^{-x} - 3x + \frac{3}{2}$

(g) $y = e^{-x}(A\sin 3x + B\cos 3x) + 1 - x$ (h) $y = A\sin 2x + B\cos 2x + \frac{3}{4}x$

(i) $y = A\sin 4x + B\cos 4x + \frac{1}{4}x^2 - \frac{1}{32}$ (j) $y = Ae^{3x} + Be^{-4x} - \frac{1}{12}x - \frac{1}{144}$

(k) $y = Ae^{-\frac{1}{2}x} + Be^{-x} - \frac{1}{2}x^2 + 3x - \frac{13}{2}$

2. (a) $y = Ae^{x} + Be^{-2x} + e^{2x}$ (b) $y = Ae^{2x} + Be^{-x} + 2e^{3x}$

(c) $y = Ae^{x} + Be^{-3x} + 2e^{2x}$ (d) $y = Ae^{x} + Be^{-2x} - 2e^{-x}$

(e) $y = (Ax + B)e^{3x} + e^{4x}$ (f) $y = e^{-x}(A\sin 2x + B\cos 2x) + \frac{1}{2}e^{3x}$

(g) $y = Ae^{-3x} + Be^{-4x} + \frac{1}{2}e^{-2x}$ (h) $y = Ae^{3x} + Be^{-2x} - \frac{1}{4}e^{-x}$

(i) $y = Ae^{x} + Be^{-3x} + \frac{1}{5}e^{2x}$ (j) $y = e^{-x}(A\sin 4x + B\cos 4x) + \frac{1}{2}e^{x}$

(k) $y = e^{-x}(A\sin 2x + B\cos 2x) + \frac{1}{8}e^{-3x}$

3. (a) $y = Ae^{x} + Be^{3x} + 4\sin x + 8\cos x$

(b) $y = Ae^{x} + Be - 2^{3x} + \sin x - 6\cos x$

(c) $y = (Ax + B)e^{-2x} + 3\sin x - 4\cos x$

(d) $y = e^{x}(A\sin x + B\cos x) - \frac{1}{5}\sin x + \frac{2}{5}\cos x$

(e) $y = e^{-x}(A\sin x + B\cos x) + \sin x - 2\cos x$

(f) $y = e^{x}(A\sin x + B\cos x) - \frac{1}{10}\sin 2x + \frac{1}{5}\cos 2x$

(g) $y = Ae^{2x} + Be^{-3x} - \frac{7}{50}\sin x - \frac{1}{50}\cos x$

(h) $y = e^{\frac{1}{2}x}(A\sin\frac{1}{2}x + B\cos\frac{1}{2}x) - \frac{4}{5}\sin x - \frac{2}{5}\cos x$

4. (a) $y = Ae^{x} + Be^{2x} - xe^{x}$ (b) $y = Ae^{2x} + Be^{3x} - 3xe^{2x}$

(c) $y = Ae^{2x} + Be^{3x} - \frac{4}{9}xe^{3x}$ (d) $y = Ae^{-3x} + Be^{-4x} + xe^{-3x}$

(e) $y = (Ax + B)e^{4x} + 3x^2e^{4x}$